

of the circle  $adce$  becomes infinite, or, what is the same thing, the current  $I_C$  is in quadrature with the voltage  $V$ . Under this condition there is but one point of resonance and it corresponds to minimum current. The conductance of the condenser circuit is zero, whereas that of the inductive branch is constant. This constant conductance makes the current at resonance a minimum, and hence the impedance a maximum. Since most selective circuits employ constant inductance and variable capacitance and the resistances of the capacitive branches are very small, maximum impedance or minimum current at resonance is practically realized in these circuits. Since at resonance the current is simply the conductance times the voltage impressed, it is evident that the power factor is 1. An inspection of Fig. 27 will reveal the manner in which the phase angle  $\theta$  between the resultant current and the applied voltage varies as the resultant current follows the circle  $adce$ . Between points  $d$  and  $e$ , leading power factor obtains.

*Resonance by Varying Frequency.* From equation (39) the frequency for parallel resonance is found to be

$$f_m = \frac{1}{2\pi\sqrt{LC}} \left[ \frac{R_L^2 C - L}{R_C^2 C - L} \right]^{1/2} \quad (40)$$

When  $R_L^2 C > L$  and  $R_C^2 C < L$ , the quantity  $\left[ \frac{R_L^2 C - L}{R_C^2 C - L} \right]^{1/2}$  is imaginary and therefore no real frequency will yield resonance. The same situation results if both inequality signs are reversed. If  $R_L$  and  $R_C$  are equal, equation (40) for resonance becomes

$$f_m = \frac{1}{2\pi\sqrt{LC}}$$

which is the same as that for series resonance. This equation is also correct when  $R_L = R_C = 0$  and may therefore be used as a close approximation when  $R_L$  and  $R_C$  are very small. It should be apparent that there are values of  $R_L$ ,  $C$ ,  $R_C$ , and  $L$  in a parallel circuit for which parallel resonance is impossible, regardless of frequency. This is in contrast to the series circuit containing  $R$ ,  $L$ , and  $C$  where there is always some real resonant frequency for any values of the three parameters. The trends of various quantities as frequency is varied from a value too small to produce resonance to a value higher than that required for resonance are shown in Fig. 28 for a condition where resonance is obtainable.

*Resonance by Varying  $R_L$  or  $R_C$ .* When equation (40) is solved for

$R_L$ , the following equations result.

$$R_L = \sqrt{\frac{LC\omega^2 (R_C^2 C - L) + L}{C}} \quad (41)$$

$$R_L = \sqrt{LC\omega^2 R_C^2 - L^2\omega^2 + \frac{L}{C}} \quad (42)$$

$$R_L = \sqrt{\frac{X_L}{X_C} R_C^2 - X_L^2 + \frac{L}{C}} \quad (43)$$

When the parameters are such as to make the expressions under the above radical positive,  $R_L$  takes on definite positive values. It is

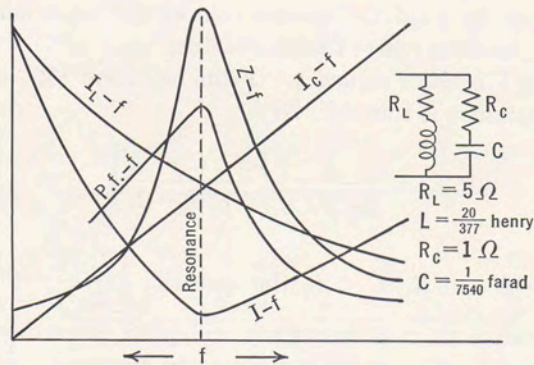


FIG. 28. Parallel resonance by varying frequency.

thus shown that within limits there are definite values of  $R_L$  which will bring the circuit to resonance at some particular values of frequency,  $L$ ,  $C$ , and  $R_C$ . Also, for resonance,

$$R_C = \sqrt{\frac{R_L^2}{\omega^2 LC} - \frac{1}{\omega^2 C^2} + \frac{L}{C}} \quad (44)$$

Equation (44) shows that, for those values of parameters which make the quantity under the radical positive, resonance may be produced by choosing the proper value of  $R_C$ .

In contrast to the series circuit, where resistances have no part in determining the frequency of resonance, the resistances of a parallel circuit are of signal importance in determining the frequency of resonance, even to the extent of making resonance either possible or impossible to attain. Physically this can be understood when it is remembered that, with a certain quadrature component of current in



the condensive branch, some sufficiently large value of  $R_L$  will prevent a resultant current in the inductive branch from flowing, which is as much as the quadrature current in the condensive circuit even when the inductance is zero. Under such conditions it is apparent that inserting inductance will do nothing but make the current in the inductive branch still smaller and hence contribute nothing toward making resonance possible. Such a case was discussed with reference to Fig. 26 when  $I_C \sin \theta_C$  was greater than  $V/2R_L$ . Figure 26, which is simply a vector diagram, shows that  $I_L \sin \theta_L$  can never be made as large as  $I_C \sin \theta_C$  if  $V/2R_L$  is less than  $I_C \sin \theta_C$ . A similar situation obtains for the condensive branch.

**Problem 8.** Draw the vector diagram and show the locus of  $I_L$  as  $X_L$  is varied, when  $R_C = 1$  ohm,  $X_C = 10$  ohms,  $R_L = 6$  ohms, and the impressed voltage 100 volts for a circuit as shown in Fig. 24. Repeat the problem when  $R_L$  is changed to 4 ohms. What is the largest possible quadrature component of current in the inductive branch as  $X_L$  is varied in each case? In which case can resonance be produced? Why?

*Ans.:* 8.33 amperes, 12.5 amperes, resonance for 4-ohm case only.

**A Simple Form of Wave Trap.** Resonance phenomena as presented in the foregoing articles form the basis upon which many circuits used in both wire and wireless communication operate. They are especially adapted to selective circuits such as those for filters and oscillators. A parallel combination of capacitance and inductance, along with its incidental resistance, can be made into an effective band eliminator, suppressor, or wave trap. The impedance of such a branch (from  $a$  to  $b$  in Fig. 29), where the resistance of the capacitance is negligibly small and  $R_L$  is very small compared to  $L$ , is most easily found by taking the reciprocal of the resultant admittance. Since the branches are tuned for parallel resonance, the resultant admittance is conductance only. Thus

$$Y_m = \frac{R_L}{Z_L^2} \quad (45)$$

and

$$Z_m = \frac{1}{Y_m} = \frac{Z_L^2}{R_L} \quad (46)$$

Since  $R_L^2 \ll \omega^2 L^2$ ,

$$Z_m = \frac{\omega^2 L^2}{R_L} \quad (47)$$

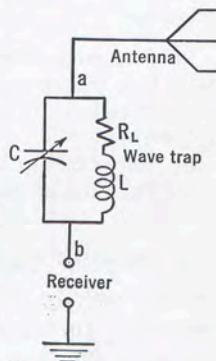


FIG. 29. Simple form of wave trap.